Fragmentation Function in Non-Equilibrium QCD Using Closed-Time Path Integral Formalism

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Abstract

In this paper we implement Schwinger-Keldysh closed-time path integral formalism in non-equilibrium QCD to the definition of Collins-Soper fragmentation function. We consider a high p_T parton in QCD medium at initial time τ_0 with arbitrary non-equilibrium (non-isotropic) distribution function $f(\vec{p})$ fragmenting to hadron. We formulate parton to hadron fragmentation function in non-equilibrium QCD in the light-cone quantization formalism. It may be possible to include final state interactions with the medium via modification of the Wilson lines in this definition of the non-equilibrium fragmentation function. This may be relevant to study hadron production from quark-gluon plasma at RHIC and LHC.

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I. INTRODUCTION

RHIC and LHC heavy-ion colliders are the best facilities to study quark-gluon plasma in the laboratory. Since two nuclei travel almost at speed of light, the QCD matter formed at RHIC and LHC may be in non-equilibrium. In order to make meaningful comparison of the theory with the experimental data on hadron production, it may be necessary to study nonequilibrium-nonperturbative QCD at RHIC and LHC. This, however, is a difficult problem.

Non-equilibrium quantum field theory can be studied by using Schwinger-Keldysh closed-time path (CTP) formalism [1, 2, 3]. However, implementing CTP in non-equilibrium at RHIC and LHC is a very difficult problem, especially due to the presence of gluons in non-equilibrium and hadronization etc. Recently, one-loop resumed gluon propagator in non-equilibrium in covariant gauge is derived in [4, 5].

High p_T hadron production at high energy e^+e^-, ep and pp colliders is studied by using Collins-Soper fragmentation function [6, 7]. For a high p_T parton fragmenting to hadron, Collins-Soper derived an expression for the fragmentation function based on field theory and factorization properties in QCD at high energy [8]. This fragmentation function is universal in the sense that, once its value is determined from one experiment it explains the data at other experiments.

The derivation of parton to hadron fragmentation function in QCD medium based on first principle calculation is not done so far. This can be relevant at RHIC and LHC heavy-ion colliders to study hadron production from quark-gluon plasma. Further complication arises because the partons at RHIC and LHC may be in non-equilibrium.

In this paper we note that, one can implement closed-time path integral formalism in non-equilibrium QCD to the definition of Collins-Soper fragmentation function. We consider a high p_T parton in medium at initial time τ_0 with arbitrary non-equilibrium (non-isotropic) distribution function $f(\vec{p})$ fragmenting to hadron. We formulate parton to hadron fragmentation function in non-equilibrium QCD in the light-cone quantization formalism. The special case $f(\vec{p}) = \frac{1}{e^{\frac{p_0}{T}} \pm 1}$ corresponds to the finite temperature QCD in equilibrium. This fragmentation function may be relevant to study hadron production from quark-gluon plasma at RHIC and LHC.

We find the following definition of the parton to hadron fragmentation function in non-

equilibrium QCD by using closed-time path integral formalism. For a quark (q) with arbitrary non-equilibrium distribution function $f_q(\vec{k})$ at initial time, the quark to hadron fragmentation function is given by

$$D_{H/q}(z, P_T) = \frac{1}{2z \; [1 + f_q(\vec{k})]} \int dx^- \frac{d^{d-2}x_T}{(2\pi)^{d-1}} e^{ik^+ x^- + iP_T \cdot x_T/z}$$

$$\frac{1}{2} \text{tr}_{\text{Dirac}} \; \frac{1}{3} \text{tr}_{\text{color}} [\gamma^+ < in|\psi(x^-, x_T) \; \Phi[x^-, x_T] \; a_H^\dagger(P^+, 0_T) a_H(P^+, 0_T) \; \Phi[0] \; \bar{\psi}(0)|in >] \quad (1)$$

where $z \; (=\frac{P^+}{k^+})$ is the longitudinal momentum fraction of the hadron with respect to the parton and P_T is the transverse momentum of the hadron. $|in >$ is the initial state of the non-equilibrium QCD medium in the Schwinger-Keldysh $in-in$ closed-time path formalism.

For a gluon (g) with arbitrary non-equilibrium distribution function $f_g(\vec{k})$ at initial time, the gluon to hadron fragmentation function is given by

$$D_{H/g}(z, P_T) = \frac{1}{2zk^+ \; [1 + f_g(\vec{k})]} \int dx^- \frac{d^{d-2}x_T}{(2\pi)^{d-1}} e^{ik^+ x^- + iP_T \cdot x_T/z}$$

$$\frac{1}{8} \sum_{a=1}^{8} [< in|F_a^{+\mu}(x^-, x_T) \; \Phi[x^-, x_T] \; a_H^\dagger(P^+, 0_T) a_H(P^+, 0_T) \; \Phi[0] \; F_{\mu a}^+(0)|in >]. \quad (2)$$

The path ordered exponential

$$\Phi[x^\mu]_{ab} = \mathcal{P} \, \exp[ig \int_0^\infty d\lambda \; n \cdot A^c(x^\mu + n^\mu \lambda) \; T_{ab}^c] \quad (3)$$

is the Wilson line [9]. It may be possible to include final state interactions with the medium via modification of the Wilson lines in this definition of the non-equilibrium fragmentation function, similar to the p_T distribution of the parton distribution function studied in [10].

Eqs. (1) and (2) can be compared with the following definition of Collins-Soper fragmentation function [6]:

$$D_{H/q}(z, P_T) = \frac{1}{2z} \int dx^- \frac{d^{d-2}x_T}{(2\pi)^{d-1}} e^{ik^+ x^- + iP_T \cdot x_T/z}$$

$$\frac{1}{2} \text{tr}_{\text{Dirac}} \; \frac{1}{3} \text{tr}_{\text{color}} [\gamma^+ < 0|\psi(x^-, x_T) \; \Phi[x^-, x_T] \; a_H^\dagger(P^+, 0_T) a_H(P^+, 0_T) \; \Phi[0] \; \bar{\psi}(0)|0 >] \quad (4)$$

and

$$D_{H/g}(z, P_T) = -\frac{1}{2zk^+} \int dx^- \frac{d^{d-2}x_T}{(2\pi)^{d-1}} e^{ik^+ x^- + iP_T \cdot x_T/z}$$

$$\frac{1}{8} \sum_{a=1}^{8} [< 0|F_a^{+\mu}(x^-, x_T) \; \Phi[x^-, x_T] \; a_H^\dagger(P^+, 0_T) a_H(P^+, 0_T) \; \Phi[0] \; F_{\mu a}^+(0)|0 >]. \quad (5)$$

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We will present derivations of eqs. (1) and (2) in this paper.

The paper is organized as follows. In section II we briefly review the definition of Collins-Soper fragmentation function in vacuum which is widely used at pp, ep and e^+e^- colliders. In section III we give a brief description of Schwinger-Keldysh closed-time path integral formalism in non-equilibrium QCD. We implement closed-time path integral formalism in non-equilibrium QCD to Collins-Soper fragmentation function in section IV. Section V contains conclusion.

II.   COLLINS-SOPER FRAGMENTATION FUNCTION IN VACUUM

In this section we will briefly review Collins-Soper fragmentation function in vacuum which is widely used at pp, ep and e^+e^- colliders. Consider a scalar gluon, for example, with four momentum k^μ in vacuum fragmenting to a hadron with four momentum P^μ. For application to collider experiments it is convenient to use light cone quantization formalism. The scalar gluon field $\phi(x)$ can be written as

$$\phi(x^-, x_T) = \frac{1}{(2\pi)^{d-1}} \int \frac{dk^+}{\sqrt{2k^+}} d^{d-2}k_T \left[e^{-ik\cdot x} a(k) + e^{ik\cdot x} a^\dagger(k) \right]_{x^+=0} \tag{6}$$

where $a^\dagger$ and a are the creation and annihilation operators respectively. $d = 4 - 2\epsilon$ where $3 - 2\epsilon$ is the space dimension. The single particle parton state is given by

$$|k^+, k_T> = a^\dagger(k^+, k_T)|0>, \qquad \text{with} \qquad a(k^+, k_T)|0> = 0 \tag{7}$$

with the normalization

$$<k^+, k_T|k'^+, k'_T> = (2\pi)^{d-1}\delta(k^+ - k'^+)\delta^{d-2}(k_T - k'_T). \tag{8}$$

Note that the correct interpretation of the state $|k>$ is created by an appropriate Fourier transform of the corresponding field operator and should not be associated with on-shell condition $k^2 = 0$ of the massless quark or gluon [12].

Consider the inclusive production of a hadron H created in the $out-$state $|H, X>$ from the parton a in the $in-$state $|k>$ with the probability amplitude

$$<H, X|k> . \tag{9}$$

The probability distribution $h_k(P)$ of the hadron with momentum P from the parton of momentum k can be found from the above amplitude. Explicitly

$$h_k(P) < k|k' > = \sum_X \; < k|H, X >< H, X|k' > = \sum_X \; < k|a_H^\dagger(P)|X >< X|a_H(P)|k' >$$
$$= < k|a_H^\dagger(P)a_H(P)|k' > \tag{10}$$

where $a_H^\dagger$ is the creation operator of a hadron H. In the light-cone quantization formalism we find (by using eqs. (7) and (8))

$$h(z, P_T) < k^+, k_T|k'^+, k'_T > = 2z(2\pi)^{d-1} D_{H/a}(z, P_T)(2\pi)^{d-1}\delta(k^+ - k'^+)\delta^{d-2}(k_T - k'_T)$$
$$= < 0|a(k^+, k_T)a_H^\dagger(P^+, P_T)a_H(P^+, P_T)a^\dagger(k'^+, k'_T)|0 > \tag{11}$$

where $D_{H/a}(z, P_T)$ is the fragmentation function and $z = \frac{P^+}{k^+}$ is the longitudinal momentum fraction of hadron H with respect to parton a. Using

$$(2\pi)^{d-1} < 0|\phi(x^-, x_T)a_H^\dagger(P^+, P_T)a_H(P^+, P_T)\phi(0)|0 > = \frac{1}{(2\pi)^{d-1}} \int \frac{dk^+}{\sqrt{2k^+}}d^{d-2}k_T \int \frac{dk'^+}{\sqrt{2k'^+}}d^{d-2}k'_T$$
$$[< 0|e^{-ik\cdot x}a(k^+, k_T)a_H^\dagger(P^+, P_T)a_H(P^+, P_T)a^\dagger(k'^+, k'_T)|0 >]_{x^+=0}$$

$$\tag{12}$$

we find from eq. (11)

$$D_{H/a}(z, P_T) = \frac{k^+}{z} \int dx^- \frac{d^{d-2}x_T}{(2\pi)^{d-1}} e^{ik^+x^- - ik_T\cdot x_T} < 0|\phi(x^-, x_T)a_H^\dagger(P^+, P_T)a_H(P^+, P_T)\phi(0)|0 > . \tag{13}$$

It is convenient to rewrite the definition in a form analogous to the definition of the distribution of partons in a hadron. The transverse momentum is of the parton relative to the hadron rather than vice versa. For this purpose we make a Lorentz transformation to a frame where the hadron's transverse momentum is zero:

$$(P^+, P_T) \rightarrow (P^+, 0)$$
$$(k^+, 0) \rightarrow (k^+, -P_T/z). \tag{14}$$

Hence we find from eq. (13)

$$D_{H/a}(z, P_T) = \frac{k^+}{z} \int dx^- \frac{d^{d-2}x_T}{(2\pi)^{d-1}} e^{ik^+x^- + iP_T\cdot x_T/z} < 0|\phi(x^-, x_T)a_H^\dagger(P^+, 0_T)a_H(P^+, 0_T)\phi(0)|0 > . \tag{15}$$

The p_T integrated fragmentation function is given by

$$d_{H/a}(z) = \int d^{d-2}P_T D_{H/g}(z, P_T)$$
$$= \frac{k^+z^{d-3}}{2\pi} \int dx^- e^{iP^+x^-/z}[< 0|\phi(x^-)a_H^\dagger(P^+, 0_T)a_H(P^+, 0_T)\phi(0)|0 >]. \tag{16}$$

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A. Quark Fragmentation Function

Following similar steps as above but performing calculation for quark we find the quark fragmentation function

$$D_{H/q}(z, P_T) = \frac{1}{2z(2\pi)^{d-1}} \int dx^- d^{d-2}x_T e^{ik^+ x^- + iP_T \cdot x_T/z}$$

$$\frac{1}{2}\text{tr}_{\text{Dirac}} \frac{1}{3}\text{tr}_{\text{color}}[\gamma^+ < 0|\psi(x^-, x_T)a_H^\dagger(P^+, 0_T)a_H(P^+, 0_T)\bar\psi(0)|0 >] \qquad (17)$$

where $\psi(x)$ is the quark field. The P_T integrated quark fragmentation function becomes

$$d_{H/q}(z) = \frac{z^{d-3}}{4\pi} \int dx^- e^{iP^+ x^-/z} \frac{1}{2}\text{tr}_{\text{Dirac}} \frac{1}{3}\text{tr}_{\text{color}}[\gamma^+ < 0|\psi(x^-)a_H^\dagger(P^+, 0_T)a_H(P^+, 0_T)\bar\psi(0)|0 >]. (18)$$

B. Gluon Fragmentation Function

Following the above steps but for gluons we find the gluon fragmentation function

$$D_{H/g}(z, P_T) = -\frac{1}{2zk^+(2\pi)^{d-1}} \int dx^- d^{d-2}x_T e^{ik^+ x^- + iP_T \cdot x_T/z}$$

$$\frac{1}{8}\sum_{a=1}^{8}[< 0|F_a^{+\mu}(x^-, x_T)a_H^\dagger(P^+, 0_T)a_H(P^+, 0_T)F_{\mu a}^+(0)|0 >] \qquad (19)$$

where

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu. \qquad (20)$$

The P_T integrated gluon fragmentation function becomes

$$d_{H/g}(z) = -\frac{z^{d-3}}{4\pi k^+} \int dx^- e^{iP^+ x^-/z} \sum_{a=1}^{8}[< 0|F_a^{+\mu}(x^-)a_H^\dagger(P^+, 0_T)a_H(P^+, 0_T)F_{\mu a}^+(0)|0 >]. (21)$$

C. Wilson Lines and Fragmentation Functions

The quark and gluon fragmentation functions as defined above are not gauge invariant. Gauge invariant parton fragmentation functions are obtained by incorporating Wilson lines $\Phi[x]_{ab}$ into the definition of the quark and gluon fragmentation functions [6, 8, 9, 11]. We find

$$D_{H/q}(z, P_T) = \frac{1}{2z(2\pi)^{d-1}} \int dx^- d^{d-2}x_T e^{ik^+ x^- + iP_T \cdot x_T/z}$$

$$\frac{1}{2}\text{tr}_{\text{Dirac}} \frac{1}{3}\text{tr}_{\text{color}}[\gamma^+ < 0|\psi(x^-, x_T) \Phi[x^-, x_T] a_H^\dagger(P^+, 0_T)a_H(P^+, 0_T) \Phi[0] \bar\psi(0)|0 >] (22)$$

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where $\Phi[x^-, x_T]$ is given by eq. (3) with T^{ab} in the fundamental representation of SU(3).

Similarly, incorporating Wilson lines, we find the gauge invariant gluon fragmentation function

$$D_{H/g}(z, P_T) = -\frac{1}{2zk^+(2\pi)^{d-1}} \int dx^- d^{d-2}x_T e^{ik^+x^- + iP_T \cdot x_T/z}$$

$$\frac{1}{8} \sum_{a=1}^{8} [< 0|F_a^{+\mu}(x^-, x_T) \ \Phi[x^-, x_T] \ a_H^\dagger(P^+, 0_T)a_H(P^+, 0_T) \ \Phi[0] \ F_{\mu a}^+(0)|0 >] \qquad (23)$$

where $\Phi[x^-, x_T]$ is given by eq. (3) with $T_c^{ab} = f^{abc}$ in the adjoint representation of SU(3).

III.   NON-EQUILIBRIUM QCD USING CLOSED-TIME PATH FORMALISM

Unlike pp collisions, the ground state at RHIC and LHC heavy-ion collisions (due to the presence of a QCD medium at initial time $t = t_{in}$ (say $t_{in}=0$) is not a vacuum state $|0 >$ any more. We denote $|in >$ as the initial state of the non-equilibrium QCD medium at t_{in}. The non-equilibrium distribution function $f(\vec{k})$ of a parton (quark or gluon), corresponding to such initial state is given by

$$< a^\dagger(\vec{k})a(\vec{k}') >=< in|a^\dagger(\vec{k})a(\vec{k}')|in >= f(\vec{k})(2\pi)^{d-1}\delta^{(d-1)}(\vec{k} - \vec{k}') \qquad (24)$$

where we have assumed space translational invariance at initial time.

Finite temperature field theory formulation is a special case of this when $f(\vec{k}) = \frac{1}{e^{\frac{k_0}{T}} \pm 1}$.

Consider scalar gluons first. In the CTP formalism in non-equilibrium there are four Green's functions

$$G++(x, x') =< in|T\phi(x)\phi(x')|in >=< T\phi(x)\phi(x') >$$
$$G--(x, x') =< in|\bar{T}\phi(x)\phi(x')|in >=< \bar{T}\phi(x)\phi(x') >$$
$$G+-(x, x') =< in|\phi(x')\phi(x)|in >=< \phi(x')\phi(x) >$$
$$G-+(x, x') =< in|\phi(x)\phi(x')|in >=< \phi(x)\phi(x') > \qquad (25)$$

where $+(-)$ sign corresponds to upper(lower) time branch of the Schwinger-Keldysh closed-time path [1, 2]. T is the time order product and $\bar{T}$ is the anti-time order product. The field $\phi(x)$ is in Heisenberg representation. Explicitly

$$T\phi(x)\phi(x') = \theta(t - t')\phi(x)\phi(x') + \theta(t' - t)\phi(x')\phi(x)$$
$$\bar{T}\phi(x)\phi(x') = \theta(t' - t)\phi(x)\phi(x') + \theta(t - t')\phi(x')\phi(x). \qquad (26)$$

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At initial time $t = t_{in} = 0$ the Heisenberg picture coincide with the Schrodinger and interaction pictures. We write

$$\phi(x) = \int \frac{d^{d-1}k}{(2\pi)^{d-1}\sqrt{2k^0}}[a(\vec{k})e^{-ik\cdot x} + a^\dagger(\vec{k})e^{ik\cdot x}] \tag{27}$$

where $a^\dagger(\vec{k})$ and $a(\vec{k})$ are creation and annihilation operators respectively. The commutation relations are given by

$$[a(\vec{k}), a^\dagger(\vec{k'})]_{t=0} = (2\pi)^{d-1}\delta^{(d-1)}(\vec{k} - \vec{k'}),$$
$$[a(\vec{k}), a(\vec{k'})]_{t=0} = [a^\dagger(\vec{k}), a^\dagger(\vec{k'})]_{t=0} = 0. \tag{28}$$

We assume space-translational invariance at initial time and find

$$[G_{ij}(x, x')]_{t=0} = [\int d^d k G_{ij}(k)e^{-ik\cdot(x-x')}]_{t=0} \tag{29}$$

where i, j are $+, -$. Explicitly

$$G_{-+}(x, x') = <\phi(x)\phi(x')> = \int \frac{d^{d-1}k}{\sqrt{2k^0}(2\pi)^{d-1}} \int \frac{d^{d-1}k'}{\sqrt{2k'^0}(2\pi)^{d-1}}$$
$$<in|[a(\vec{k})e^{-ik\cdot x} + a^\dagger(\vec{k})e^{ik\cdot x}][a(\vec{k'})e^{-ik'\cdot x'} + a^\dagger(\vec{k'})e^{ik'\cdot x'}]|in>. \tag{30}$$

Using Bogolyubov transformation we can set

$$<in|a(\vec{k})a(\vec{k'})|in> = <in|a^\dagger(\vec{k})a^\dagger(\vec{k'})|in> = 0. \tag{31}$$

By using eqs. (24), (28) and (31) in eq. (30) we find

$$[G_{-+}(x, x')]_{t=0} = [\int \frac{d^{d-1}k}{2k^0(2\pi)^{d-1}}[[1 + f(\vec{k})]e^{-ik\cdot(x-x')} + f(\vec{k})e^{ik\cdot(x-x')}]_{t=0}$$
$$= [\int \frac{d^d k}{(2\pi)^{d-1}}\delta(k^2)e^{-ik\cdot(x-x')}[\theta(k_0)[1 + f(\vec{k})] + \theta(-k_0)f(-\vec{k})]_{t=0} = [\int d^d k G_{-+}(k)e^{-ik\cdot(x-x')}]_{t=0}. \tag{32}$$

Similarly

$$[G_{+-}(k)]_{t=0} = \delta(k^2)[\theta(-k_0) + \theta(k_0)f(\vec{k}) + \theta(-k_0)f(-\vec{k})]$$
$$[G_{++}(k)]_{t=0} = \frac{1}{k^2 + i\epsilon} + \delta(k^2)[\theta(k_0)f(\vec{k}) + \theta(-k_0)f(-\vec{k})]$$
$$[G_{--}(k)]_{t=0} = \frac{-1}{k^2 - i\epsilon} + \delta(k^2)[\theta(k_0)f(\vec{k}) + \theta(-k_0)f(-\vec{k})]. \tag{33}$$

A. Quarks in non-Equilibrium

The non-equilibrium (massless) quark propagator at initial time $t = t_{in}$ is given by (suppression of color indices are understod)

$$G(k)_{ij} = \not{k} \begin{pmatrix} \frac{1}{k^2+i\epsilon} + 2\pi\delta(k^2)f_q(\vec{k}) & -2\pi\delta(k^2)\theta(-k_0) + 2\pi\delta(k^2)f_q(\vec{k}) \\ -2\pi\delta(k^2)\theta(k_0) + 2\pi\delta(k^2)f_q(\vec{k}) & -\frac{1}{k^2-i\epsilon} + 2\pi\delta(k^2)f_q(\vec{k}) \end{pmatrix} \quad (34)$$

where where $i,j = +, -$ and $f_q(\vec{k})$ is the arbitrary non-equilibrium distribution function of quark.

B. Gluons in Non-Equilibrium

We work in the frozen ghost formalism [4, 5] where the non-equilibrium gluon propagator at initial time $t = t_{in}$ is given by (the suppression of color indices are understood)

$$G^{\mu\nu}(k)_{ij} = -i[g^{\mu\nu} + (\alpha - 1)\frac{k^\mu k^\nu}{k^2}] \, G^{vac}_{ij}(k) - iT^{\mu\nu}G^{med}_{ij}(k) \quad (35)$$

where $i,j = +, -$. The transverse tensor is given by

$$T^{\mu\nu}(k) = g^{\mu\nu} - \frac{(k \cdot u)(u^\mu k^\nu + u^\nu k^\mu) - k^\mu k^\nu - k^2 u^\mu u^\nu}{(k \cdot u)^2 - k^2} \quad (36)$$

with the flow velocity of the medium u^μ. $G^{\mu\nu}_{ij}(k)$ are the usual vacuum propagators of the gluon

$$G^{vac}_{ij}(k) = \begin{pmatrix} \frac{1}{k^2+i\epsilon} & -2\pi\delta(k^2)\theta(-k_0) \\ -2\pi\delta(k^2)\theta(k_0) & -\frac{1}{k^2-i\epsilon} \end{pmatrix} \quad (37)$$

and the medium part of the propagators are given by

$$G^{med}_{ij}(k) = 2\pi\delta(k^2)f_g(\vec{k}) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \quad (38)$$

IV.   FRAGMENTATION FUNCTION IN NON-EQUILIBRIUM QCD USING CLOSED-TIME PATH FORMALISM

For simplicity, let us consider the scalar gluon first in the light cone quantization formalism. Generalizing the vacuum analysis in [6] we define the state $|k^+, k_T >$ of the fragmenting gluon in non-equilibrium QCD medium at initial time $x^+ = x^+_{in}$ (say at $x^+_{in} = 0$)

$$|k^+, k_T >= a^\dagger(k^+, k_T)|in > . \quad (39)$$

The non-equilibrium distribution function $f(k^+, k_T)$ of the fragmenting gluon is given by

$$< in|a^\dagger(k^+, k_T)a(k'^+, k'_T)|in >= f(k^+, k_T)(2\pi)^{d-1}\delta(k^+ - k'^+)\delta^{(d-2)}(k_T - k'_T) \qquad (40)$$

where we have assumed the space (x^- and x_T) translational invariance at initial time $x^+ = x^+_{in}$. The commutation relations are given by

$$[a(k^+, k_T), a^\dagger(k'^+, k'_T)]_{x^+=0} = (2\pi)^{d-1}\delta(k^+ - k'^+)\delta^{(d-2)}(k_T - k'_T),$$
$$[a(k^+, k_T), a(k'^+, k'_T)]_{x^+=0} = [a^\dagger(k^+, k_T), a^\dagger(k'^+, k'_T)]_{x^+=0} = 0 \qquad (41)$$

By using eqs. (24) and (28) we find

$$< k^+, k_T|k'^+, k'_T >=< in|a(k^+, k_T)a^\dagger(k'^+, k'_T)|in >$$
$$= (2\pi)^{d-1}\delta(k^+ - k'^+)\delta^{d-2}(k_T - k'_T)[1 + f(k^+, k_T)]. \qquad (42)$$

Consider the inclusive production of hadron H created in the $out-$state $|H, X >$ from a scalar gluon in non-equilibrium in the initial state $|k >$ with the probability amplitude

$$< H, X|k > . \qquad (43)$$

X being other outgoing final state particles. Similar to the vacuum case of Collins-Soper fragmentation function, the correct interpretation of the above state $|k >$ is created by an appropriate Fourier transform of the corresponding field operator and should not be associated with on-shell condition $k^2 = 0$ of the massless quark or gluon [12]. The distribution $h_k(P)$ of the hadron H with momentum P from the parton of momentum k can be found from the above amplitude. We find

$$\sum_X < k, k_T|H, X >< H, X|k^{+'}, k'_T >= h_k(P) < k^+, k_T|k^{+'}, k'_T > . \qquad (44)$$

For the left hand side we write

$$\sum_X < k^+, k_T|H, X >< H, X|k^{+'}, k'_T >= \sum_X < k^+, k_T|a_H^\dagger(P)|X >< X|a_H(P)|k^{+'}, k'_T >$$
$$=< k^+, k_T|a_H^\dagger(P)a_H(P)|k^{+'}, k'_T > . \qquad (45)$$

Equating eqs. (44) and (45) and by using eq. (39) we find

$$< in|a(k^+, k_T)a_H^\dagger(P^+, P_T)a_H(P^+, P_T)a^\dagger(k'^+, k'_T)|in >$$
$$= 2z(2\pi)^{d-1}D_{H/a}(z, P_T) < in|a(k^+, k_T)a^\dagger(k'^+, k'_T)|in > . \qquad (46)$$

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This expression is exactly similar to that of the jet fragmentation function in vacuum as given in eq. (11) except that the vacuum expectation is replaced by medium average at the initial time $x^+ = x_{in}^+$. Using eqs. (39) and (40) we find

$$< in|a(k^+, k_T)a_H^\dagger(P^+, P_T)a_H(P^+, P_T)a^\dagger(k'^+, k'_T)|in >= 2z(2\pi)^{d-1}D_{H/a}(z, P_T) \left[1 + f(k^+, k_T)\right].$$
(47)

From eq. (27) we obtain

$$(2\pi)^{d-1} < in|\phi(x^-, x_T)a_H^\dagger(P^+, P_T)a_H(P^+, P_T)\phi(0)|in >= \frac{1}{(2\pi)^{d-1}} \int \frac{dk^+}{\sqrt{2k^+}}d^{d-2}k_T \int \frac{dk'^+}{\sqrt{2k'^+}}d^{d-2}k'_T$$

$$[< in|e^{-ik\cdot x}a(k^+, k_T)a_H^\dagger(P^+, P_T)a_H(P^+, P_T)a^\dagger(k'^+, k'_T)|in >]_{x^+=0}.$$
(48)

Using this in eq. (47) we find the expression of the fragmentation function in non-equilibrium QCD

$$D_{H/a}(z, P_T) = \frac{k^+}{z \left[1 + f(k^+, k_T)\right]} \int dx^- \frac{d^{d-2}x_T}{(2\pi)^{d-1}} e^{ik^+ x^- - ik_T \cdot x_T}$$

$$< in|\phi_a(x^-, x_T)a_H^\dagger(P^+, P_T)a_H(P^+, P_T)\phi_a(0)|in > .$$
(49)

From eq. (14) we find

$$D_{H/a}(z, P_T) = \frac{k^+}{z \left[1 + f(k^+, k_T)\right]} \int dx^- \frac{d^{d-2}x_T}{(2\pi)^{d-1}} e^{ik^+ x^- + iP_T \cdot x_T/z}$$

$$< in|\phi_a(x^-, x_T)a_H^\dagger(P^+, 0_T)a_H(P^+, 0_T)\phi_a(0)|in > .$$
(50)

In the above expression, $f(k^+, k_T)$ is the non-equilibrium distribution function of the fragmenting gluon at initial time $x^+ = x_{in}^+$ and $|in >$ is the initial state of the non-equilibrium QCD medium in the Schwinger-Keldysh $in - in$ closed-time path formalism.

A. Quark Fragmentation Function in Non-Equilibrium

Following the above steps, but for quarks, we find the quark fragmentation function in non-equilibrium QCD

$$D_{H/q}(z, P_T) = \frac{1}{2z \left[1 + f_q(k^+, k_T)\right]} \int dx^- \frac{d^{d-2}x_T}{(2\pi)^{d-1}} e^{ik^+ x^- + iP_T \cdot x_T/z}$$

$$\frac{1}{2}\text{tr}_{\text{Dirac}} \frac{1}{3}\text{tr}_{\text{color}}[\gamma^+ < in|\psi(x^-, x_T)a_H^\dagger(P^+, 0_T)a_H(P^+, 0_T)\bar{\psi}(0)|in >$$
(51)

where $f_q(k^+, k_T)$ is the non-equilibrium distribution function of the fragmenting quark at initial time.

B. Gluon Fragmentation Function in Non-Equilibrium

For gluons we consider frozen ghost formalism described above (35). Hence all the analysis of the above can be applied. Carrying out the similar algebra as above we find the gluon fragmentation function

$$D_{H/g}(z, P_T) = -\frac{1}{2zk^+ \left[1 + f_g(k^+, k_T)\right]} \int dx^- \frac{d^{d-2}x_T}{(2\pi)^{d-1}} e^{ik^+x^- + iP_T \cdot x_T/z}$$
$$\frac{1}{8} \sum_{a=1}^{8} [< in|F_a^{+\mu}(x^-, x_T) a_H^\dagger(P^+, 0_T) a_H(P^+, 0_T) F_{\mu a}^+(0)|in >] \tag{52}$$

where $f_g(k^+, k_T)$ is the non-equilibrium distribution function of the fragmenting gluon at initial time.

C. Wilson Lines

The above parton to hadron fragmentation function definition in non-equilibrium QCD is not gauge invariant. To make it gauge invariant we need to incorporate Wilson lines. Incorporating the Wilson lines (eq. (3)) into the definition of the fragmentation function eq. (51) we find the gauge invariant quark fragmentation function in non-equilibrium QCD

$$D_{H/q}(z, P_T) = \frac{1}{2z \left[1 + f_q(k^+, k_T)\right]} \int dx^- \frac{d^{d-2}x_T}{(2\pi)^{d-1}} e^{ik^+x^- + iP_T \cdot x_T/z}$$
$$\frac{1}{2} \text{tr}_{\text{Dirac}} \frac{1}{3} \text{tr}_{\text{color}} [\gamma^+ < in|\psi(x^-, x_T) \ \Phi[x^-, x_T] \ a_H^\dagger(P^+, 0_T) a_H(P^+, 0_T) \ \Phi[0] \ \bar{\psi}(0)|in >] \tag{53}$$

where $f_q(k^+, k_T)$ is the non-equilibrium distribution function of the fragmenting quark at initial time $x^+ = x_{in}^+$. $|in >$ is the initial state of the non-equilibrium QCD medium in the Schwinger-Keldysh $in - in$ closed-time path formalism. This reproduces eq. (1).

Incorporating the Wilson lines (eq. (3)) into the definition of the fragmentation function eq. (52) we find the gauge invariant gluon fragmentation function in non-equilibrium QCD

$$D_{H/g}(z, P_T) = -\frac{1}{2zk^+ \left[1 + f_g(k^+, k_T)\right]} \int dx^- \frac{d^{d-2}x_T}{(2\pi)^{d-1}} e^{ik^+x^- + iP_T \cdot x_T/z}$$

$$\frac{1}{8} \sum_{a=1}^{8} [< in|F_a^{+\mu}(x^-, x_T) \ \Phi[x^-, x_T] \ a_H^\dagger(P^+, 0_T) a_H(P^+, 0_T) \ \Phi[0] \ F_{\mu a}^+(0)|in >] \quad (54)$$

where $f_g(k^+, k_T)$ is the non-equilibrium distribution function of the fragmenting gluon at initial time $x^+ = x_{in}^+$. $|in >$ is the initial state of the non-equilibrium QCD medium in the Schwinger-Keldysh $in - in$ closed-time path formalism. This reproduces eq. (2).

It may be possible to include final state interactions with the medium via modification of the Wilson lines in this definition of the non-equilibrium fragmentation function, similar to the p_T distribution of the parton distribution function studied in [10].

V.   CONCLUSIONS

In this paper we have implemented closed-time path integral formalism in non-equilibrium QCD to the definition of Collins-Soper fragmentation function. We have considered a high p_T parton in QCD medium at initial time τ_0 with arbitrary non-equilibrium (non-isotropic) distribution function $f(\vec{p})$ fragmenting to hadron. We have formulated parton to hadron fragmentation function in non-equilibrium QCD in the light-cone quantization formalism. This may be relevant to study hadron production from quark-gluon plasma at RHIC and LHC. It may be possible to include final state interactions with the medium via modification of the Wilson lines in this definition of the non-equilibrium fragmentation function, similar to the p_T distribution of the parton distribution function studied in [10]. This will be the subject of a future analysis.

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